

Demands calibration in water distribution networks by a Hybrid Method

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The tendency of the state variables (head and flow) to be monitored remotely in several points of a water supply system (as a consequence of the acquisition and transmission equipment cost reduction) has been motivating companies to incorporate routines for calibration so as to ensure that input data in the simulation model are compatible to real data. Such routines are based, in general, on the errors of minimisation of state variables. These are obtained by the differences between the monitored values and the ones calculated through a hydraulic model. This procedure relapses into a minimum search process of a non-linear function. Several methods for a solution have been proposed, of which characteristics should be analysed with the hydraulic model responsible for the evolution of the objective function. The paper describes a new model for demands calibration based on the junction between the hydraulic model TMA (time marching approach) and the searching algorithm of Nelder-Mead. The general proceedings of the proposition and an example are presented.

Introduction

According to Eggener and Polkowski (1976), there has been an upsurge of interest in the impact of expedients commonly employed for network simulation, i.e. skeletonisation, load consolidation, and pipe roughness value allocation, with increased emphasis on digital computer modelling of the hydraulic performance of water distribution networks for design and automatic operational control. Detailed models of real systems have been developed. It is very important to verify their behaviour under various conditions of loading, adjust them to obtain an overall best fit, and then use them as research tools to examine the impact of simulation expedients and the model's sensitivity to various other assumptions. Generally, network simulation models are integrated with data from the office, designs, field excursions, etc. However, the results are sometimes just compared with head and flow field measurements. Therefore, the model accuracy degree is unknown and it is impossible to assure the quality of obtained results. Thus, projects based in models formed in that way, can show a great degree of uncertainty.

Water networks calibration

Some definitions:

“Calibration consists of determining the physical and operational characteristics of an existing system and the data, which after being input to the computer model, will yield realistic results.” (Shamir, Howard, 1977).

“Calibration of a water distribution model is a two-step process consisting of: (1) comparison of pressures and flows predicted with observed pressures and flows for known operating conditions (i.e., pump operation, tank levels, pressure-reducing valve settings), and (2) adjustment of the input data for the model to improve agreement between observed and predicted values.” (Walski, 1983).

“The process of fine-tuning a model until it simulates field conditions for a specific time horizon (such as maximum hour conditions) to an established degree of accuracy.” (Cesario & Davis, 1984).

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Ormsbee and Lingireddy (1997) state that, in general, a network model calibration effort should encompass seven basic steps:

- (1) Identifying the intended use of the model.
- (2) Determining initial estimates of the model parameters.
- (3) Collecting calibration data.
- (4) Evaluating the model results.
- (5) Performing the macro-level calibration.
- (6) Performing the sensitivity analysis.
- (7) Performing the micro-level calibration.

According to these authors, deviations between results of the model application and the field observations may be caused by several factors, including:

- Erroneous model parameters (pipe roughness values and nodal demand distribution).
- Erroneous network data (pipe diameters, lengths, etc.).
- Incorrect network geometry (pipes connected to the wrong nodes).
- Incorrect pressure zones boundary definitions.
- Errors in boundary conditions (incorrect pressure-regulating valve settings, tank water levels, pump curves, etc.).
- Errors in historical operating records (i.e., pumps starting and stopping at the wrong times).
- Measurement equipment errors (i.e., pressure gauges not properly calibrated).
- Measurement error (i.e., reading the wrong values from measurement instruments).

Proposition of a Hybrid Model

Liggett (1993) and Luvizotto Jr. (1998) presented inverse methods to determine leaks in water supply systems for steady and unsteady flows. These are the methods also required for model calibration. An inverse problem is one where the output is known from measurements (i.e., heads) and where the system characteristics producing that output (i.e., flows, pipe roughness) are found. In general, inverse methods are carried out jointly with a simulator (direct) and an optimiser to minimise the equation known as objective function (OF):

$$\min \sum_{i=1}^N (E^* - E(c))_i^2$$

where $i = 1$ to N monitored nodes, E^* : monitored effects (head and/or flow) and $E(c)$: effects calculated by the model. To make possible the application of inverse methods on leaks detection, alternative techniques for problem solution were investigated. The same ideas are now used for calibrate demands distribution in water networks. The result is the use of hydraulic model, with flexible formulation and solution through the method of characteristics. The model advantages were described by Luvizotto Jr., in 1995. Shimada (1992) gave the name to the method: time marching approach, known as TMA. This method is joined to a procedure of function minimisation, related in 1965, by Nelder and Mead, and the procedure has kept their name. This linkage between the two described techniques forms the Hybrid Method. According to Solomatine (1998), many issues related to water resources require the solution of linear and non-linear optimisation techniques, which normally assume that the minimised function (objective function) is known in analytical form and that it has a single minimum. In practice, however, there are many problems that cannot be described analytically and many objective functions have multiple extrema, where optimisation methods are non-applicable and other solutions must be investigated. According to Wright (1996), this situation of generating functions, of which the derivatives are unknown or non-existent, arises in many contexts, particularly in real-world applications, what shows the importance of the method here described, due to its non-derivative characteristics.

The Nelder-Mead Algorithm

Barton and Ivey Jr. (1996) described the method as follows:

1. **INITIALIZATION.** For a function of n parameters, choose $n + 1$ extreme points to form an initial n -dimensional simplex. Evaluate the response function $F(x_i)$ at each extreme point (vertex) x_i of the simplex for $i = 1, 2, \dots, n + 1$.

2. **STOPPING CRITERION.** Iterations continue until the standard deviation of the $n + 1$ response function values at the extreme points of the simplex,

$$S_F \equiv [\sum (F(x_i) - \bar{F})^2 / (n + 1)]^{1/2},$$

with $\bar{F} \equiv \sum F(x_i) / (n + 1)$, falls below a particular value, or when the size of the simplex becomes sufficiently small, or until the maximum number of function evaluations is reached.

3. **REFLECT WORST POINT.** At the start of each iteration, identify the vertices where the highest, second highest, and lowest response function values occur. Let P_{high} , P_{sechi} , P_{low} respectively denote these points, and let F_{high} , F_{sechi} , F_{low} respectively represent the corresponding observed function values. Find P_{cent} , the centroid of all vertices other than P_{high} . Generate a new vertex P_{refl} by reflecting P_{high} through P_{cent} . Reflection is carried out according to the following equation, where α is the reflection coefficient ($\alpha > 0$):

$$P_{\text{refl}} = (1 + \alpha)P_{\text{cent}} - \alpha P_{\text{high}}.$$

Nelder and Mead used $\alpha = 1$.

4a. **ACCEPT REFLECTION.** If $F_{\text{low}} \leq F_{\text{refl}} \leq F_{\text{sechi}}$, then P_{refl} replaces P_{high} in the simplex, and a new iteration begins (step 2 above).

4b. **ATTEMPT EXPANSION.** If $F_{\text{refl}} < F_{\text{low}}$, then the reflection is expanded, in the hope that more improvement will result by extending the search in the same direction. The expansion point is calculated by the following equation, where the expansion coefficient is γ ($\gamma > 1$).

$$P_{\text{exp}} = \gamma P_{\text{refl}} + (1 - \gamma)P_{\text{cent}}.$$

Nelder and Mead used $\gamma = 2$. If $F_{\text{exp}} < F_{\text{low}}$, then P_{exp} replaces P_{high} in the simplex; otherwise, the expansion is rejected and P_{refl} replaces P_{high} . The next iteration begins with the new simplex (step 2 above).

4c. **ATTEMPT CONTRACTION.** If the reflected vertex has the worst response function value in the new simplex (i.e., $F_{\text{refl}} > F_{\text{sechi}}$), then the simplex contracts. If $F_{\text{refl}} \leq F_{\text{high}}$, then P_{refl} replaces P_{high} and F_{refl} replaces F_{high} before attempting contraction or shrinking. The contraction vertex is calculated by the following equation, where the contraction coefficient is β ($0 < \beta < 1$):

$$P_{\text{cont}} = \beta P_{\text{high}} + (1 - \beta)P_{\text{cent}}.$$

Nelder and Mead used $\beta = 0.5$. If $F_{\text{cont}} \leq F_{\text{high}}$, then contraction is accepted and the algorithm continues with the next iteration (step 2 above).

4c'. **SHRINK.** If $F_{\text{cont}} > F_{\text{high}}$, then the contraction has failed, and the entire simplex shrinks by a factor of δ ($0 < \delta < 1$), retaining only P_{low} . This is done by replacing each extreme point P_i (except P_{low}) by:

$$P_i \leftarrow \delta P_i + (1 - \delta)P_{\text{low}}.$$

Nelder and Mead used $\delta = 0.5$. The algorithm then evaluates F at each vertex (except P_{low}) and continues with the next iteration (step 2 above).

One or more of the three stopping criteria are usually employed. Nelder and Mead computed the standard deviation of the (deterministic) objective function values over all $n + 1$ extreme points, and they stopped when the standard deviation S_f dropped below 10^{-8} , where

$$S_f \equiv [\sum (f(x_i) - \bar{f})^2 / (n + 1)]^{1/2},$$

with $\bar{f} \equiv \sum f(x_i) / (n + 1)$. For stochastic functions, the standard deviation of function values across all simplex vertices reflects inherent stochastic variation as well as differences in (expected) function values. For stochastic function optimization, the calculations are based on F rather than f , and the stochastic component ϵ in equation (2) will typically have a standard deviation σ , $\equiv [E(\epsilon^2)]^{1/2}$ much greater than 10^{-8} , making this rule inappropriate: the standard deviation S_f of the F -values computed over the simplex vertices is an inaccurate estimator of the standard deviation S_f of the f -values computed over the simplex vertices, particularly as the simplex decreases in size near the optimum and S_f approaches zero.

A stopping criterion based on simplex size was proposed by Dennis and Woods (1987). The stopping criterion is

$$(1/\Delta) \max_i \|P_i - P_{\text{low}}\| \leq \nu, \quad \Delta = \max(1, \|P_{\text{low}}\|), \quad (4)$$

where the maximization is over all extreme points i in the current simplex, and $\|\cdot\|$ denotes the Euclidean norm.

Application

In order to show the performance of the Hybrid Method, an example is presented. The nodes 3, 5, 9 and 12 are considered pressure monitoring stations and nodes 4, 7, 8 and 10 the points of nodal demands. Data shown in the figure and tables are supposed to be the real ones. When initialising the procedure, the real values of nodal demands are changed to false ones so that the model finds the correct demands “by itself”.

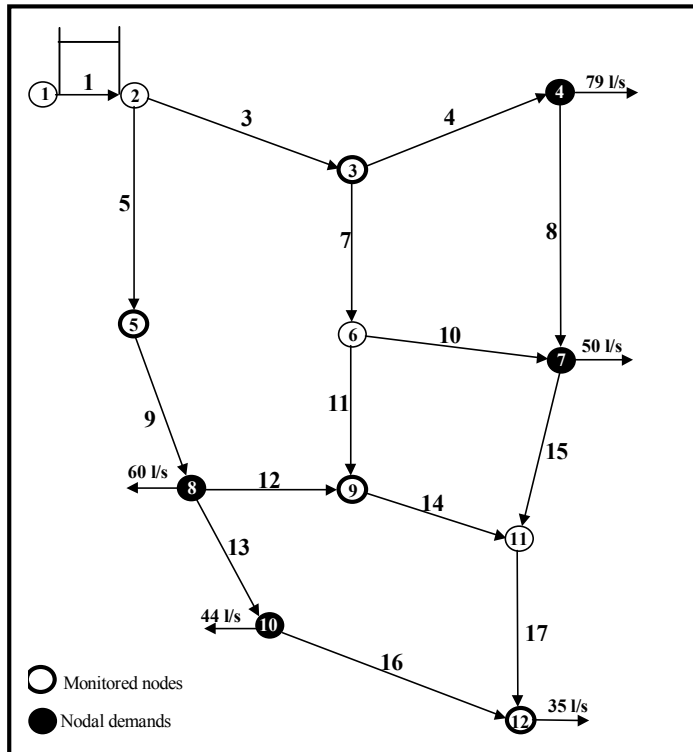


Fig. 1 Network geometry and data.

PIPES			
Pipe	L (m)	D (mm)	C
3	3000	500	100
4	1500	400	100
5	2500	400	100
7	1500	200	100
8	2200	300	100
9	1500	400	100
10	2200	500	100
11	2500	200	100
12	2700	200	100
13	2000	250	100
14	1700	200	100
15	1800	250	100
16	1000	200	100
17	1200	200	100

RESERVOIR	
Res.	LEVEL (m)
1	100

MONITORED HEADS (wcm)	
Node	Head
3	94,26
5	91,71
9	85,56
12	73,23

As there are 4 nodes to have the demands calibrated, the Nelder-Mead procedure uses 5 (equal to “n+1”) arrays of nodal demands. These arrays, selected randomly, are shown in table 1.

Table 1- Initial random arrays of nodal demands (l/s)

Arrays of demands	Demand nodes			
	4	7	8	10
1	75	45	57	40
2	75	60	57	40
3	90	45	57	40
4	75	45	72	40
5	75	45	57	55

Results and conclusions

The convergence process has been successful in finding the correct values of nodal demands. It was necessary 45 iterations and 396 network solutions to reach the desired data, as shown in Fig. 2.

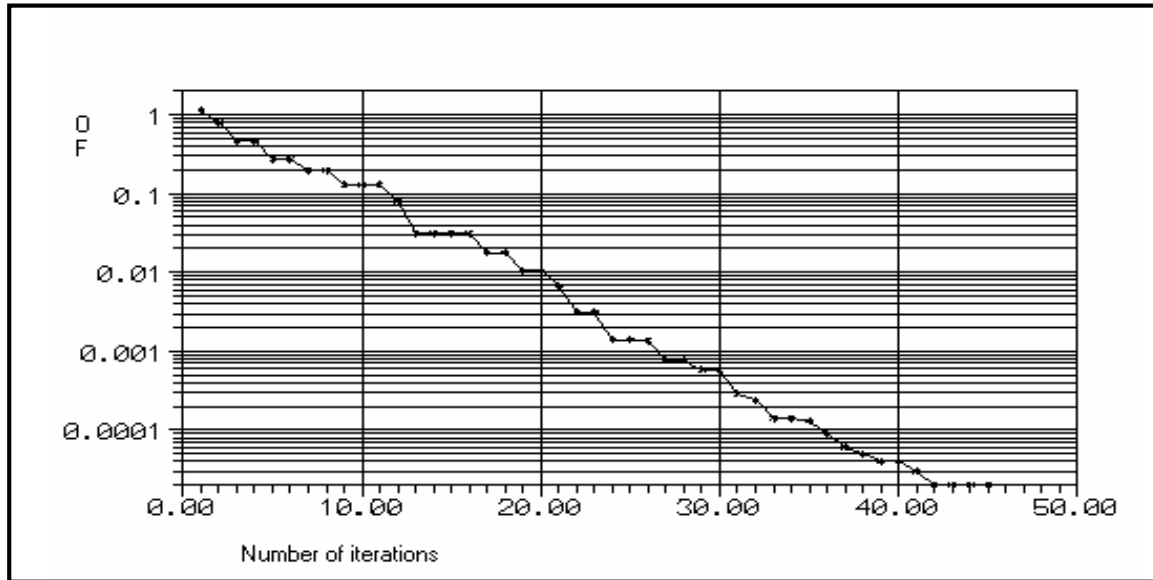


Fig. 2 – Convergence of objective function (OF)

The Nelder-Mead algorithm has performed very well when coupled to a simulator based on TMA, and, though it has been used just values of head, it's simple to define an mixed objective function, containing also values of flow.

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