Free-Surface Fluid Flows over Spillway

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Abstract

The analysis of two-dimensional fluid flows with a free surface has been of interest to hydrodynamicists for a long period of time. The flow characteristics of special interest include the velocity distribution, pressure distribution (over the entire flow field), discharge coefficient, contraction coefficient, and the free surface location. This paper focuses on two-dimensional, irrotational flows of a homogeneous, inviscid, incompressible fluid with a free surface boundary. The objective of this paper was to develop a method to achieve a prescribed accuracy and analyze a wide variety of irrotational flow problems.

The geometry of the spillway ogee crest has been modified to satisfy the tangency condition with the downstream face of the dam. The finite element model (FEM) was developed and applied to obtain numerical solutions of the flow problems with respect to headwater conditions. In this study, the effect of the geometric modification of the spillway ogee crest was assessed to verify or update the spillway discharge capacity. The numerical solution for each case yields the free-surface profile of the free-over flow and flow discharge. The computed water-surface profiles are similar to those obtained from the U.S. Army Corps of Engineers' test data (USACE, 1988).

Keywords : fluid, flow, free-surface, spillway, FEM, dam

1 Introduction

The analysis of two-dimensional ideal fluid flows involving a free-surface has attracted the attention of hydrodynamicists for a very long time. Flow characteristics of interest may include the velocity distribution, or pressure distribution or both, over the entire flow field, and also the discharge coefficient, contraction coefficient, and the free-surface location. To date exact solutions for these quantities have still not been found for many problems of practical interest. Most of the existing methods can solve only problems with simple geometric boundaries.

This study focused attention on two-dimensional, irrotational flow of a homogeneous, inviscid, incompressible fluid which has a free surface boundary. The goal was to develop a method which could achieve a prescribed accuracy and could be used to analyze a wide variety of irrotational problems, assuming the ideal fluid theory is valid. To examine the applicability and versatility of the method, it was used to analyze flows over spillway. In spillway, the stability of the free surface flow is dependent on the geometry of the approach channel as well as that of the spillway itself. It is therefore very important to design the geometry to minimize the free surface waves to avoid choking or overflow. This is a very difficult task that usually requires a costly model-testing program (Song et al. 1999). The physical experiments for the spillway flows are studied by Ippen (1951), Reinauer and Hager (1998). Also, there are some numerical models for a spillway flows in literature (Ellis and Pender 1982; Montes 1994; Ghan and Steffler 1996).

As part of the review and update of the previous design documents for the Boyabat Dam, the geometry of the spillway ogee crest has been modified by Harza to satisfy the tangency condition at the point of tangency with the downstream face of the dam. The slope of the downstream face has been changed from 0.82H : 1V to 0.75H : 1V slope. The purpose of this study is to assess the spillway capacity on the Boyabat Dam. All spillway gates of the dam were assumed to be fully opened.

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2 Description

The Boyabat dam is a conventional concrete gravity-type dam. It is 185-m high from the existing river bed elevation, with a crest elevation of 335.0-m above sea level. The total height of the dam from the foundation is 195 meters. The spillway crest is at El. 318.0 m, and consists of three chutes controlled by six gates.

The upstream face of the dam is vertical, and the slope of the downstream face is 0.75H: 1V. The full supply level (maximum normal water level) in the reservoir is 330 meters above sea level, while the minimum normal water level is 305 meters above sea level during normal operation. The gross storage capacity is 3,557 x 10^6 m^3 and the effective storage capacity is 1,410 x 10^6 m^3.

The project design peak inflow is 9,300 m^3/sec, which results in a surcharge of the reservoir pool to a headwater elevation of 334.2 m and spillway outflow 8,300 m^3/sec. The estimated probable maximum flow at the dam site is about 12,300 m^3/sec. The spillway, which will be located on the dam body, consists of six radial gates with cent overflow and three spillway chutes. Each gate has a width of 10.0-meter and a height of 12.0-meter. The upper part of the chute spillway slopes at the same inclination as the downstream face of the dam, and the lower portion just before the flip bucket is at an angle of 8 degrees. The flip bucket lip is at an angle of 20-degree, discharging spillway flow away from the powerhouse structure, located beneath the spillway.

3 Finite Element Modeling

3.1 Governing Equations and Boundary Conditions

The flow approaching and passing over the spillway ogee crest converges along its flow paths, and can be adequately described mathematically by the potential flow equations with appropriate boundary conditions prescribed. The flow is assumed incompressible, and irrotational. The governing flow equation is

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

in which f is the velocity potential function, and x and y are respectively the Cartesian coordinates in the horizontal and vertical directions. The velocity potential function is defined such that

$$u = -\frac{\partial f}{\partial x}, \quad v = -\frac{\partial f}{\partial y}$$

where u and v are the x and y velocity components.

The free surface is a free-streamline and is subject to a constant atmospheric pressure such that

$$\frac{p}{g} = 0$$

where p is pressure and g is the specific weight of water. Along the free surface, the streamline tangency condition must be satisfied, that is, the normal velocity components must be zero.

$$\frac{\partial f}{\partial n} = 0$$

The incoming flow through the upstream boundary is assumed to be uniformly distributed such that

$$\frac{\partial f}{\partial n} = -\underline{n}$$

where $\underline{n}$ is the incoming velocity of the flow, and *n* is the unit outward normal. To satisfy the flow uniformity assumption, the upstream boundary is located at a sufficient distance upstream from the upstream face of the dam.

The downstream boundary of the flow model is considered as a constant potential line. In the analysis, an arbitrary value of potential is assigned to this boundary, i.e.

$$f = const.$$

The Bernoulli energy equation is applied throughout the entire flow region such that

$$H = \frac{v^2}{2g} + \frac{p}{g} + y$$

where *H* is the total head at the upstream boundary of the model with respect to the bottom of the reservoir at the dam, n is velocity and *y* is elevation above the bottom of the reservoir at the dam. The total head is assumed constant throughout the entire flow region of the model. Alternately, the pressure coefficient (C_p) can be computed, defined as

$$C_p = \frac{p - p_d}{\frac{1}{2} g v^2}$$

in which p_d = reference pressure (usually atmospheric).

3.2 Finite Element Procedure

The basic finite element analysis procedure applied to solve the ideal fluid flow problem can be found (Zienkiewicz 1971). The governing partial differential equation of the ideal fluid potential flow is converted into finite element equations through the variational principle. The flow region to be analyzed is subdivided into eight-node quadrilateral elements, and each element consists of four triangular sub-elements. Each triangular sub-element has three corner nodes and three mid-side nodes to facilitate quadratic representation of the velocity potential in each element. Such arrangement allows the velocity components to vary linearly within each element. The resulting system of linear equations is solved simultaneously for nodal velocity potentials by the Gaussian elimination procedure.

3.3 Free Surface Treatments

The problem of flow over the spillway ogee crest involves a free surface and is difficult to solve since the location of the free surface is not known initially. There are two boundary conditions that must be satisfied over the free surface as described previously. The procedure (Wei and DeFazio 1981) applied to adjust the free surface profile consists of the following steps:

(1) An initial free surface profile is assumed.
(2) Based on the constant pressure condition and the Bernoulli equation, the velocity potential is calculated for each free surface node starting from the downstream and where an arbitrary value of the velocity potential is assigned.
(3) Form the system matrix for nodal velocity potentials through the finite element procedure. The system matrix is modified to incorporate the boundary conditions.
(4) The system equations are solved for nodal velocity potentials, and velocity components are calculated for all free surface nodes.
(5) The re-calculated velocity vectors are applied to re-locate the free surface with a spline curve fitting procedure.

These five steps constitute a basic computation cycle. At the end of the fourth step, the solution accuracy is checked, and the computation can be terminated when the calculated downstream outflow satisfies the continuity criterion and the total head based on the recalculated nodal velocity at each free surface node satisfies the total head specified at the upstream boundary of the model to a desired degree of accuracy. In the analysis, the computation is terminated when the computed outflow is less than 0.2 percent of the inflow.

4 Results

As part of the review and update of the previous design documents for the Boyabat Dam, the geometry of the spillway ogee crest has been modified by Harza to satisfy the tangency condition at the point of tangency with the downstream face of the dam. The equation describing the spillway ogee crest has been changed from

$y = 0.0446363x^{1.864}$ to $y = 0.0468700x^{1.860}$

The slope of the downstream chute face was also changed from 0.82H: 1V to 0.75H: 1V.

In this analysis, the effect of the geometric modification of the spillway ogee crest on the spillway discharge capacity was assessed to verify or update the spillway discharge capacity. The changes are relatively small especially at the upper portion of the ogee crest where the geometric effect on the discharge capacity is more pronounced.

The spillway capacity of the original EPDC design was determined based on a hydraulic model study of the Boyabat Dam. The original design would yield the following discharges for the design flood (El. 334.2m) and the normal maximum (El. 330.0m) water surface levels.

Table 1. Two reservoir headwater conditions and its discharges

Water Surface Elevation, m		Head Difference, m (=Water Surface Elevation - Dam Appex Elevation)	Discharge, m^3/sec	
			Original Design	Modified Design
Design Flood	334.2	16.2	8,300.0	8,304.0
Normal Maximum	330.0	12.0	5,000.0	4,993.0

In table 1, the above two reservoir headwater conditions were used to determine the spillway discharges based on the modified ogee crest geometry. For each case, a finite element model was constructed using the modified spillway ogee crest geometry. The water surface level for the design flood is 334.2m, and the appex of the dam is located at El. 318m. The normal maximum water surface level is located at El. 330.0m, and composed of 202 grids. The finite element method was then applied to obtain numerical solutions of the flow problems with respect to the above two headwater conditions; design flood and normal maximum water surface levels. Since the objective of this study is to verify the discharge capacity of the spillway during flooding, all spillway gates were assumed to be fully opened. The governing flow equation, the treatment of the boundary conditions, and the numerical solution procedure are described in Section 3 entitled “Finite Element Modeling”.

The numerical solution for each case yields the free-surface profile of the free-over flow and the flow discharge for the dam. The computed water surface profiles are obtained from the numerical simulation. The two water levels are almost parallel at the downstream of the dam. The results simulated by finite element model are compared with the U.S. Army Corps of Engineers (USACE) test data for the design floodwater surface level. Also, the simulated and USACE’s data for the normal maximum water level are also obtained using the developed FEM model. These profiles are similar to those obtained from the USACE’s test data (US Army Corps of Engineer 1988).

The velocity distribution over the spillway is shown in Figure 1. In Figure 1, the maximum velocity of downstream spillway is about 22.5 m/sec. Since the direction of the flow is given by the streamlines as shown in Figure 2, the flow net gives an excellent quantitative picture of the flow. In Table 1, the computed discharges are 8,304 m^3/sec and 4,993 m^3/sec for headwater level at El. 334.2m and El. 330.0m respectively. The difference between the computed discharges for the revised spillway geometry and the original discharges are less that 0.15 percent of the original estimated discharges, and can be considered as negligible. The results indicate that the effect on the discharge capacity is negligible.

5 Conclusion

To examine the applicability and versatility of the method, it was used to analyze flows over spillway. The spillway capacity of a dam is the most important to protect its safety during a flood. During the finalization of the Boyabat Dam Spillway design, the original ogee crest geometry was modified to improve its transition and asymtoticity to the downstream chute slope.

The finite element model was developed and applied to obtain numerical solutions of the flow problems with respect to two headwater conditions; the design flood and normal maximum water surface levels for the Boyabat Dam in Turkey. In this analysis, the effect of the geometric modification of the spillway ogee crest on the spillway discharge capacity is assessed and the locations (or the profiles) of free surface is founded. The results indicate that the effect on the discharge capacity is negligible owing to the geometric modification of the spillway ogee crest.

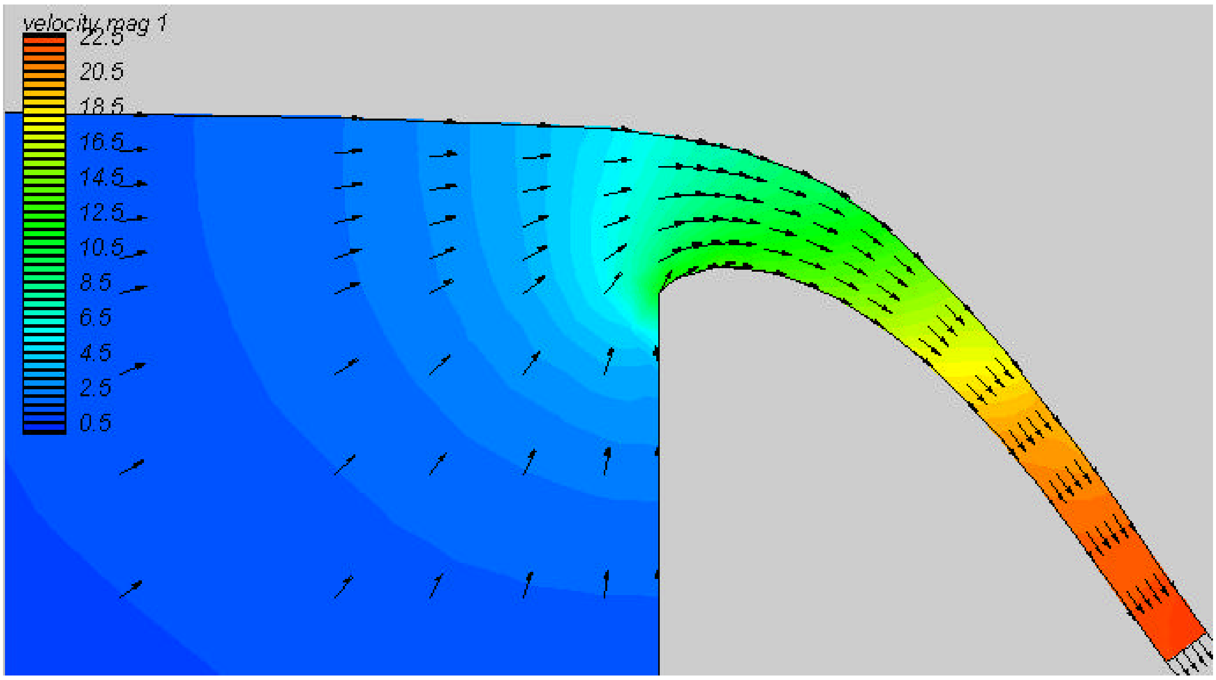

Figure 1. Velocity Distribution over the Spillway (Unit: m/sec)

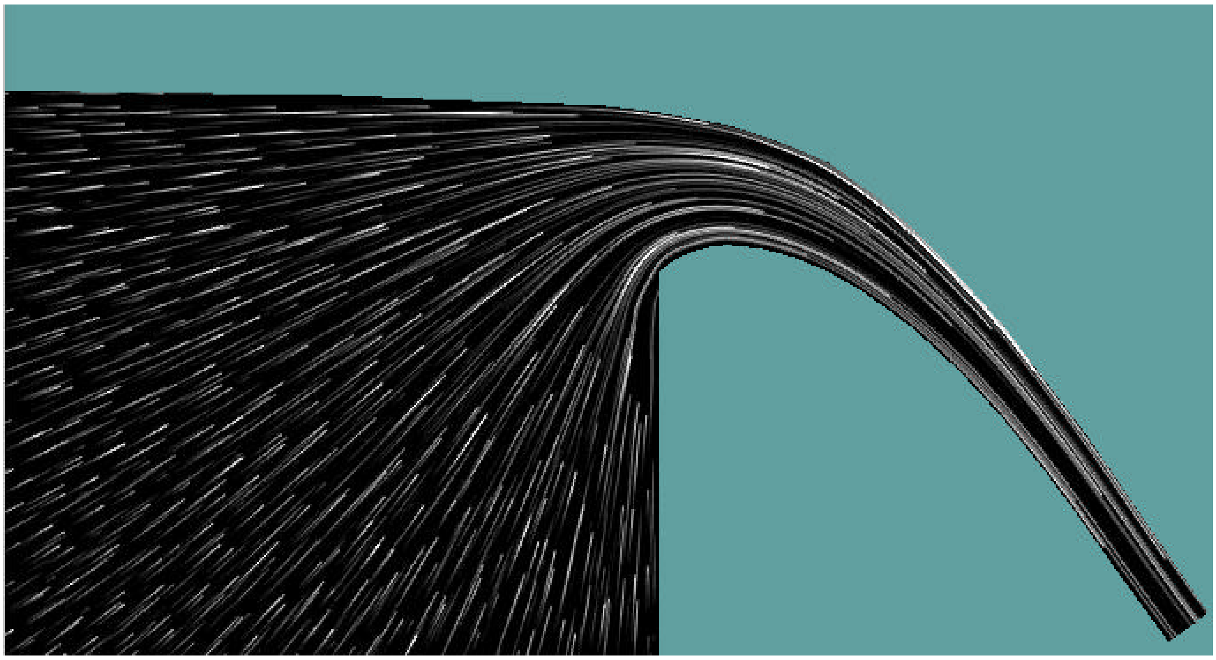

Figure 2. Streamlines over the Spillway

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